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The effect of slope algorithms on slope estimates within a GIS

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ABSTRACT

Digital elevation models (DEMs) form an important part of many geographic information system (GIS) datasets; equally important are the parameters calculated from these DEMs. This paper addresses the currently available methods of calculating slope angle from a digital elevation model and introduces a new method which circumvents a number of the shortcomings associated with other algorithms. The results of the comparison of four different slope angle calculation algorithms show that maximum downhill slope angle calculations retain the local variability present in the original DEM without overestimating slopes.

INTRODUCTION

The maximum downhill slope angle algorithm proposed by Hickey *et al* (1994) is statistically compared to three existing slope algorithms. The improvement in accuracy for deriving slope within a geographic information system (GIS) will benefit a wide range of environmental models because slope attributes are frequently needed as input. For example, computer models for bushfires, landslides, land planning and construction all use slope angle as input.

Erosion models, in particular, rely heavily on the accuracy of slope angle calculations for estimating erosion. The areal non-point source watershed environmental response simulation model (ANSWERS) (Beasley and Huggins, 1982), the agricultural non-point source pollution model (AGNPS) (Young *et al*, 1985), the water erosion prediction project (WEPP) (Foster and Lane, 1987), the Universal Soil Loss Equation (USLE) (Wischemeier and Smith, 1978), and the Revised Universal Soil Loss Equation (RUSLE) (Renard *et al*, 1991) are examples of erosion models which use slope angle as a required input. These models divide a watershed into a grid and assign environmental attributes such as soil type, slope angle, rainfall, and slope length to each cell. Grid-based watershed models can be easily interfaced with a GIS. A GIS can extract slope angles from digital elevation models (DEM) using the slope algorithms compared in this report.

All the above erosion models can take advantage of the automated calculation of slope angles from a DEM using any of the algorithms compared in this report. However, the effects of the slope algorithms on slope angle estimation (and, therefore, predicted erosion) can vary greatly in accuracy.

METHODS USED TO PREDICT SLOPE

Slope maps are often used as layers within a GIS and can display major differences depending on the algorithm used for their derivation. The four methods described below have been used extensively for slope prediction within GIS. The first two, the neighbourhood and quadratic surface algorithms, calculate an average across the centre cell using at least four of the surrounding eight cells. The second two, the maximum slope and the maximum downhill slope algorithms calculate the slope angle based upon the centre cell and one of the surrounding eight neighbours. The single difference between the two is that the maximum slope algorithm selects the maximum slope angle from the absolute value of the slopes between the centre cell and the eight surrounding cells; the maximum downhill slope angle algorithm does not use the absolute value. As such, when using the maximum downhill slope algorithm, uphill slopes are assigned a negative value -- clearly a (numerically) lower slope than anything in a downhill direction. Figure 1 illustrates the cell numbering system used for the range of slope angle algorithms considered in this research.

Neighbourhood Method

This technique is employed by both Arc/Info GRID (ESRI, 1995) and GRASS (Geographical Resources Analysis Support System) (CERL, 1988) and uses a moving three by three mask over a DEM to predict slope for the centre cell from its eight neighbours. The equation for slope (rise/run ratio) of the centre cell (#9) (percent slope) is:

$$S = (\sqrt{(S^2_{e\text{-}w} + S^2_{n\text{-}s})}) * 100 \qquad (1)$$

east-west slope is given by:

$$S_{e\text{-}w} = \frac{(z_3 + 2z_4 + z_5) - (z_1 + 2z_8 + z_7)}{4*2*d} \qquad (2)$$

north-south slope is given by:

$$S_{n\text{-}s} = \frac{(z_1 + 2z_2 + z_3) - (z_7 + 2z_6 + z_5)}{4*2*d} \qquad (3)$$

where: S = slope ratio in percent

z_1 to z_9 = elevations of cells 1 to 9

d = cell resolution

The neighbourhood method does not consider the elevation at the centre of the three by three mask, which leads to inaccurate estimations of slope if the elevation data have small pits, peaks, or if the mask is centred along a ridge or valley. Therefore, a smoothing filter is often used on the elevation data before calculating the slope angles to eliminate small pits and peaks (Srinivasan and Engel, 1991). Even if not initially smoothed, the output from this algorithm effectively smoothes the slope surface. While this leads to a loss of the local variability in the output slope map, it may be desirable in cases where the initial DEM is inaccurate. In addition, any significant changes in flowdirection across a cell may give misleading slope values, primarily because such a surface can no longer be adequately described by a plane. For example, if the flow into the centre cell is from the west and the flow from the centre cell is to the south, this algorithm can produce inappropriate results.

Quadratic Surface Method

A partial quadratic equation can be used to pass exactly through the nine elevation points in a three by three mask (Zevenbergen and Thorne, 1987). The slope (in percent) is the first derivative of z with respect to the direction of slope, which is given by:

$$S = (\sqrt{(G^2 + H^2)}) * 100 \qquad (4)$$

$$G = \frac{-z_8 + z_4}{2*d} \qquad (5)$$

$$H = \frac{z_2 - z_6}{2*d} \qquad (6)$$

where: S = slope ratio in percent

z_2, z_4, z_6 and z_8 = elevations of cells 2, 4, 6 and 8

d = cell resolution

This method considers only the 4 adjacent neighbours (z_2, z_4, z_6 and z_8) of the centre cell (z_9) and is, therefore, limited in its consideration of the local variability surrounding the centre cell. In addition, the same limitations inherent in the neighbourhood method also apply to the quadratic surface algorithm.

Maximum Slope Method

This method, unlike the previous two methods, considers the elevation of the centre cell (z_9) when estimating the slope. Shanholtz *et al* (1990) proposed that the maximum slope (rise/run ratio) between the centre cell (z_9) and its eight neighbours (z_1 to z_8) would be used as the estimate for slope at the centre cell in the three by three mask. IDRISI for Windows (IDRISI, 1997) uses this algorithm, with the exception that only four adjacent cells (N,S,E,W) are considered. The expression for maximum slope (in percent) is:

$$S = \max \frac{|(z_9 - z_i)|}{L_c} * 100 \qquad (7)$$

where: S = slope ratio in percent

L_c = distance between neighbouring cell midpoints (cell resolution) ($d*\sqrt{2}$ for neighbours diagonally adjacent to the centre cell (z_1, z_3, z_5 and z_7)).

$i = 1, 2, 3, \ldots 8$

The primary disadvantage of the maximum slope method is the overestimation of slopes. When the mask moves across the elevation grid, slope values equal in magnitude, but opposite in sign, will be computed for adjacent cells as each is the centre of its mask. Effectively, a steep slope will be counted twice, once for a cell in an uphill direction and again for the adjacent cell in the downhill direction. Therefore, slope angles are overestimated for that area of terrain.

Maximum Downhill Slope Method

This method is similar to maximum slope but does not compute the absolute value of the difference between the centre cell (z_9) and its neighbours (z_1 to z_8). Instead, Hickey *et al* (1994) computes the maximum of the downhill slope values of a three by three mask using the following formula:

$$S = \max \frac{(z_9 - z_i)}{L_c} * 100 \qquad (8)$$

where: S = slope ratio in percent

L_c= distance between neighbouring cell midpoints (d*√2 for neighbours diagonally adjacent to the centre cell (z_1, z_3, z_5 and z_7).

i = cell 1, 2, 3,...8

The maximum downhill slope method corrects the flaws in the maximum slope method because there is no overestimation of slope caused by copying of absolute maximum differences between elevation cells. When compared to the averaging algorithms, the primary advantage is that the local variability is retained by considering only the centre cell and one neighbour. Therefore, in summary, local variability is retained without overestimating slopes.

COMPARISON OF METHODS FOR SLOPE PREDICTION

Methodology

The four slope angle algorithms were compared using an eight by eight grid of elevation data (Figure 2a) within Microsoft Excel. To eliminate any edge effect errors associated with not having eight surrounding cells, output slope values for the outer two rows/columns were dropped from consideration. Thus, only 36 slope values were used in the comparisons. The elevation grid was designed such that the western area is relatively flat (24 cells) and the eastern area is relatively steep (12 cells). A valley roughly bisects the two regions. The grid cells have a resolution of 100 metres.

A DEM was derived from the elevation grid using IDRISI for Windows version 2.0 (Figure 2b). For visualisation purposes, the DEM was smoothed several times with a mean filter to eliminate some of the blocky appearance of the elevation grid.

Each of the slope algorithms was executed in Microsoft Excel using the formulae mentioned earlier. The slope angles calculated by each method were analysed statistically (mean and standard deviation) according to the flat, steep and complete DEM sections.

Slope Angle Analysis

As a general statement, the four slope angle algorithms can be classified as either averaging or non-averaging. The neighbourhood and quadratic surface algorithms are averaging algorithms because four or more cells in a mask are used to calculate the slope of the centre cell. The maximum slope and maximum downhill slope algorithms are non-averaging and calculate the slope between the centre cell and a single neighbouring cell.

As shown in Table 1, average slopes calculated by the maximum downhill slope, neighbourhood, and quadratic surface algorithms are similar; those calculated by the maximum slope algorithm are always higher. The maximum slope method gives an average slope value for the whole DEM about 1.7 times greater than the other methods (Figure 3a). This is caused by steep slopes being counted twice, once for a cell in an uphill direction and once again for the adjacent cell in the downhill direction. As can be seen from the graphs of average slope angle results for the flat and steep areas (Figures 3b and 3c), the maximum slope method shows an average slope about 1.8 times greater than the other methods for the flatter area as opposed to about 1.3 times greater for the steeper area. These results are similar to the average slope results shown by Srinivasan and Engel's (1991) comparison of the maximum slope, quadratic surface, and neighbourhood algorithms.

The standard deviations for the averaging algorithms are low compared to both the maximum slope and maximum downhill slope calculations (Figures 3a, 3b and 3c). This is because these algorithms calculate an average slope across four or more cells in the mask as opposed to calculating the slope between only two cells. In the steeper regions, this averaging results in lower average slopes than the maximum downhill slope angle output; this trend is reversed (for the same reasons) in the flat regions.

Figure 4 is a sample DEM which illustrates a region (nine 3x3 neighbourhoods, centre cells shown in bold) that would be affected by a single higher elevation value on a flat surface. The cell resolution is set at 10 metres for simplicity in interpretation. Figures 5a, 5b, 5c, and 5d illustrate the effects of the different slope algorithms upon this DEM

(Figure 4); output is in percent slope. As can be seen, the effects are highly variable. The averaging algorithms smooth the effects of the single cell over a large area (Figures 5a and 5b), but do not consider the centre cell -- in this case, the only cell that has a variable elevation. The maximum slope algorithm indicates high slopes throughout the area (Figure 5c), while the maximum downhill slope calculations (Figure 5d) limit the effects of the single high point to only one cell. It should be noted that this is an extreme, not typical, example of the variable results possible using different slope angle algorithms.

CONCLUSIONS

Since it's beginning in the 1960's, GISs have been used to assist the management of large tracts of the earth's surface. A common theme in these management plans at nearly every scale has been a model of the topography. Given this, an accurate estimation of the topography and topographic attributes is essential. However, a number of different models, with significantly different outputs, have been developed to describe topographic attributes, particularly slope, within a GIS. A better understanding of the effects of the different models is necessary to improve GIS-based descriptions of the earth's surface and, therefore, improve the management plans which are based upon these descriptions.

This paper addresses four commonly used algorithms available for calculating slope within a GIS. Each has advantages and disadvantages. The averaging techniques lose local variability, do not consider the centre cell, and have inherent problems associated with peaks, pits, ridges, and valleys. However, the smoothing effects may be entirely appropriate when DEM accuracy is suspect and a general impression may be more valuable than local variations. The maximum slope algorithm retains the local variability inherent in the DEM, but tends to dramatically overestimate slopes. Finally, the maximum downhill slope angle algorithm retains the local variability, but, as a result of constraining calculations to the downhill direction, does not overestimate slopes.

To build upon this research, future analyses could compare observed (measured in the field) slope angles with neighbourhood, quadratic surface, maximum slope and maximum downhill slope methods. Again flat and steep terrain could be used to compare results and a digital elevation model could be derived on a landscape or watershed scale which covers several different land uses. Central tendency and dispersion could be measured to evaluate which algorithm gives the closest estimate to the observed slope angle. In addition, another weak area in slope modelling which requires testing is the effect of scale on the analysis. At what point are improvements in DEM resolution irrelevant? At the other end of the scale, at what resolution are calculations of slope too general to be meaningful?

In conclusion, perhaps the most important lesson learned in this study is that one cannot take for granted the algorithms provided by different software packages. Even something as common and straightforward as slope angle can (and is) calculated in very different ways. As a result, any analysis which includes a slope angle component will be biased by the assumptions inherent in the slope algorithm. A greater understanding of the algorithms used should lead to better interpretations of analyses which require accurate descriptions of the earth's surface.

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Captions

Table 1. *Statistics for the calculated slope angles.*

Figure 1. *3x3 Mask schematic.*

Figure 2a. *Elevation grid (100m grid cells with values in metres).*

Figure 2b. *DEM - vertical exaggeration 1.5x.*

Figure 3a. *Graph of average slope - Whole DEM.*

Figure 3b. *Graph of average slope - Flat region.*

Figure 3c. *Graph of average slope - Steep region.*

Figure 4. *Sample DEM (resolution 10 metres) illustrating the area (nine 3x3 neighbourhoods, centre cells highlighted in bold) affected by a single peak.*

Figure 5a. *Results of neighbourhood slope calculations on sample DEM.*

Figure 5b. *Results of quadratic surface slope calculations on sample DEM.*

Figure 5c. *Results of maximum slope calculations on sample DEM.*

Figure 5d. *Results of maximum downhill slope calculations on sample DEM.*

	Whole Region (64 cells)		Flat Region (24 cells)		Steep Region (12 cells)	
	Avg	Std Dev	Avg	Std Dev	Avg	Std Dev
Neighbourhood	8.25	2.94	6.96	2.08	10.82	2.77
Quadratic Surface	8.69	4.24	7.29	3.50	11.49	4.32
Maximum Slope	14.58	5.77	13.42	5.64	16.9	5.54
Maximum Downhill	8.47	6.08	6.1	4.05	13.22	6.81
Mean	10.00		8.44		13.11	
Standard Deviation	3.06		3.35		2.72	

Table 1.

1	2	3
8	9	4
7	6	5

Figure 1.

80	65	75	55	40	75	100	135
75	70	80	45	35	55	90	125
70	60	45	30	30	80	65	100
65	55	45	35	25	65	50	75
50	40	50	30	20	45	60	80
65	45	30	25	15	45	55	70
50	35	25	20	10	30	45	50
35	25	25	20	10	30	30	35

Figure 2a.

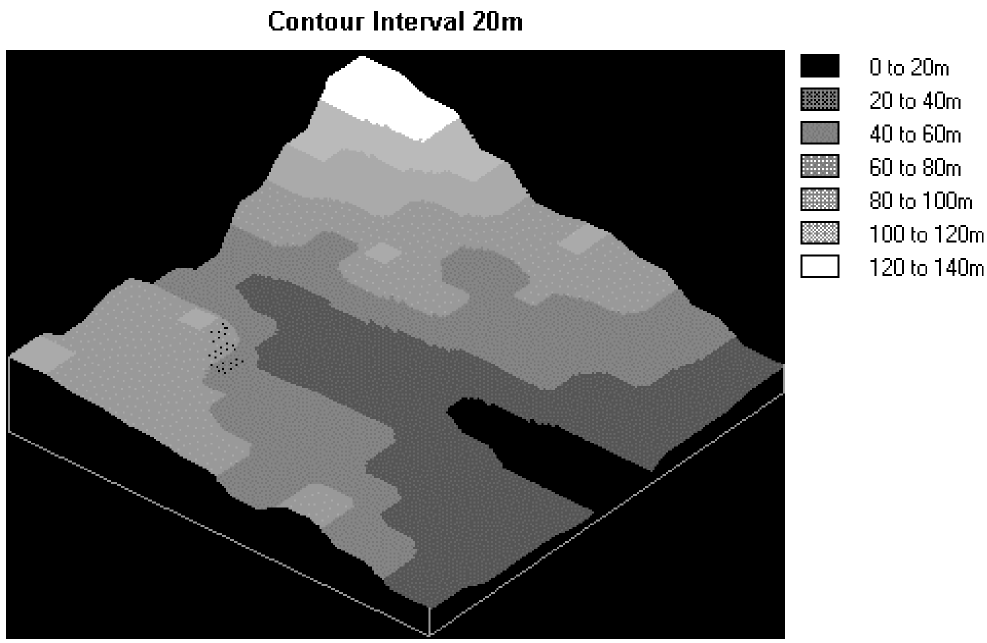

Figure 2b.

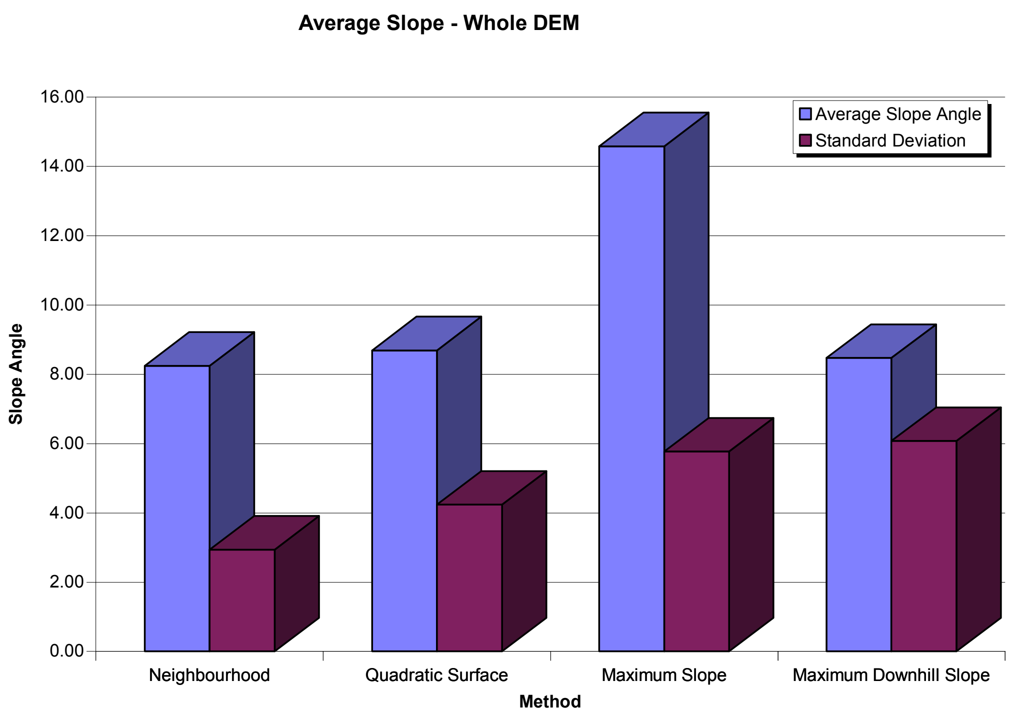

Figure 3a.

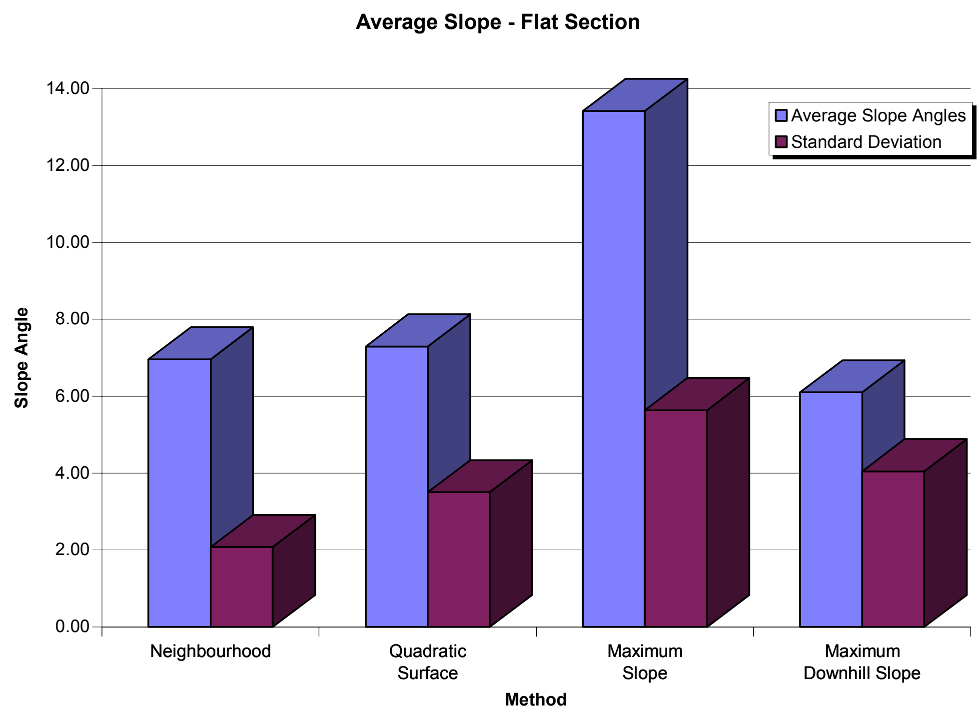

Figure 3b.

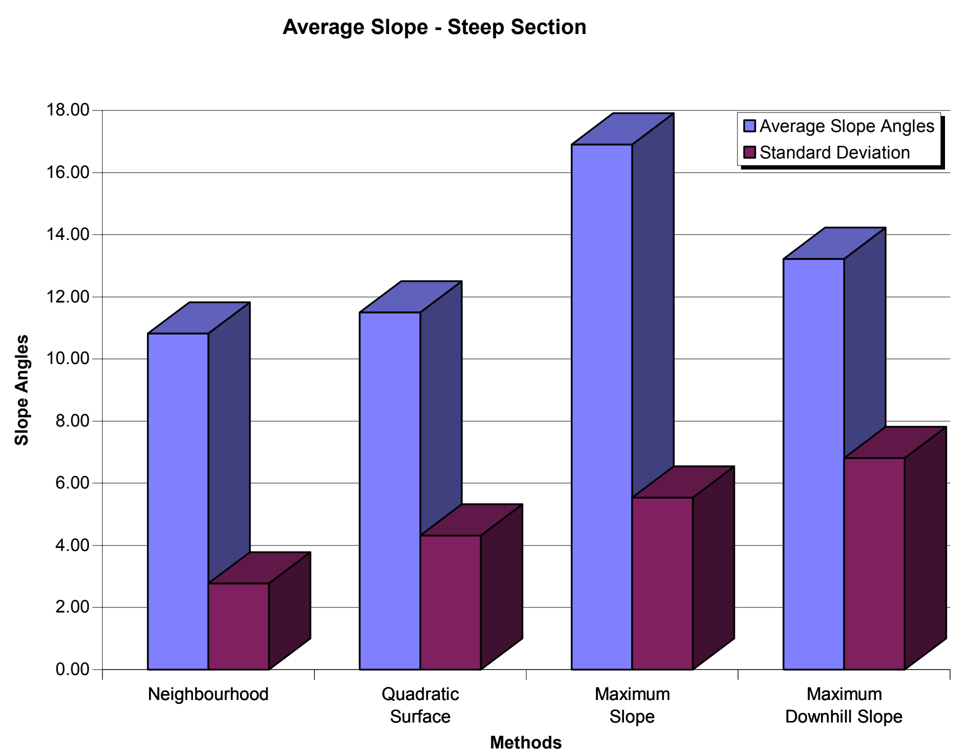

Figure 3c.

90	90	90	90	90
90	**90**	**90**	**90**	90
90	**90**	**100**	**90**	90
90	**90**	**90**	**90**	90
90	90	90	90	90

Figure 4.

18%	35%	18%
35%	0%	35%
18%	35%	18%

Figure 5a.

0%	50%	0%
50%	0%	50%
0%	50%	0%

Figure 5b.

71%	100%	71%
100%	100%	100%
71%	100%	71%

Figure 5c.

0%	0%	0%
0%	100%	0%
0%	0%	0%

Figure 5d.